A Condition-Based Inspection-Maintenance Model Based on Geometric Sequences for Systems With a Degradation Process and Random Shocks

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Abstract

In this paper we present a condition-based inspection-maintenance model for the systems subject to a degradation process and random shocks based on a geometric approach where the inter-inspection times are non-increasing. Upon the inspection of the system and its condition, inspectors need to decide whether to take an action such as preventive maintenance (PM) or corrective maintenance (CM), or no action is needed. We first derive an expected maintenance cost rate function subject to a degradation process and cumulative shock damage. Then we use the mean time to first failure as an initial solution approach to obtain the optimal maintenance policy consisting of two decision variables that minimizes the expected long-run maintenance cost per unit time. The two decision variables are the preventive maintenance threshold value and inspection times based on a geometric sequence. A numerical example is given to illustrate the optimal maintenance policy of the expected long-run cost rate model.

Keywords- Condition-based maintenance, cumulative random shock damage, maintenance optimization, preventive maintenance, expected long-run cost rate, degradation process, geometric inspection sequence

1. Introduction

Suppose that the state of a system can only be revealed through the inspection. This research we study an optimal inspection-maintenance strategy for repairable systems subject to degradation and random shocks. Grall et al [1] study the inspection-maintenance strategy for a single unit deteriorating system. The degradation is expressed by the Gamma process which implies the stationary and independent increment property. The maintenance cost function is developed based on regenerative and semi-regenerative process. Preventive and inspection schedule are two decision variables. While, in this research, it is assumed that the degradation and shock damage are measurable, otherwise there are some parameters associated with the processes which can be traced. The maintenance decision is made on the amount of degradation and shock damage which are measured, not on something abstract such as the distribution parameters or transition probability.

Grall, Berenguer and Dieulle [2] consider a system subject to a random deterioration process. They develop a model that allow to investigate the joint influence of the preventive maintenance threshold and inspection dates based on the average long-run cost rate assuming that the degradation process is a stationary law. In this research, we do not assume the stationary degradation process but the degradation follows the degradation paths and hope that, it is more practical. Pham et. al. [21] present a degradation model for predicting the reliability of k-out-of-n systems based on a Markov approach in which components are subject to multi-stage degradation and catastrophic failures. The same authors [22] later study models for predicting the availability and mean lifetime of multi-stage degraded systems with partial repairs. They, however, have not considered the maintenance aspects in their studies. Wang and Pham [23] recently develop a dependent competing risk model for systems subject to multiple degradation processes and random shocks using time-varying copulas. Their model considers a flexible dependence relationship between random shocks and degradation processes as well as the dependent relationship among various degradation processes.

So [6] studies the control limit policies for a multistate deteriorating system which is modeled by a semi-Markov process with a state space {0,...,M}. A control limit policy is a policy such that when the system condition is worse than a certain threshold n...
Suppose that the state of a system can only be revealed consisting of two decision variables that minimizes the approach to obtain the optimal maintenance policy an expected maintenance cost rate function subject to a non-increasing. Upon the inspection of the system and its A Condition-Based Inspection-Maintenance I.

(i) When the state of a system revealed by an inspection is as maintenance. In this research, two maintenance actions are based on the degradation paths instead of the conditional with dependent competing risks of degradation wear and generally, maintenance is classified into condition-based or time-based. For the former, the action taken depends on the state of the system detected after each inspection, which could be determined to perform a PM, CM or nothing. For the CM, the maintenance action is performed at predetermined time intervals to bring the system to an improved working condition and would prefer as the condition-based maintenance. In this research, two maintenance actions are considered: preventive and corrective maintenance. The system is instantaneously inspected by time \( I_1, \ldots, I_n \). At inspection time \( I_i \), one of the two decisions has to be made:

(1) Determine whether the maintenance action is PM, CM, or the system as is, and

(2) Determine the time to next inspection.

In other words,

(i) When the state of a system revealed by an inspection is as \( Y(t) \leq L \cap D(t) \leq S \) the system is left as it is since the system is at very health condition.

(ii) When the system state upon the inspection is found as \( L < Y(t) \leq G \cap D(t) \leq S \), the functioning system is considered as worn-out and a PM is triggered.

When the system fails caused by either degradation or random shocks, a CM is taken to restore the system to as good as new. PM is an active action to avoid the failure of a system during the actual operation since the cost and damage when the system failure is often large. \( L \) is called PM threshold level which is a control limit value. When the system state deteriorates to or beyond \( L \), the system is preventively maintained.

It is assumed that no continuous monitoring is performed on the system. So the state of the system is only revealed after each inspection. The choice of the inspection times \( \{I_1, \ldots, I_n\} \) and PM threshold level \( L \) have great influence on the maintenance cost rate. \( L \) effectively divides the system state into two sets. On the one hand, a low \( L \) values will result in a frequently PM action and prevents the full usage of the residual life of the systems. Frequent PMs might reduce the chances of high deterioration and failures but it also costly. On the other hand, a high \( L \) values will keep the system working in a high risk
condition. For the condition-based maintenance, the regular inspection (equal inter-inspection) is more convenient to schedule. Although the losses due to down time can be reduced by frequent inspections, it might not always worthwhile to inspect the unit, especial if the inspection is expensive. Sequential inspection is more realistic. Li and Pham [20] recently discuss a condition-based maintenance modeling aspect in which the system is periodically inspected at an increasing equally time of intervals such as 1, 2I, 3l, ..., nl where l is the first inspection time interval. A reason for such approach is that in some applications today, the preventive maintenance threshold is likely to be set conservatively and the inspection schedule may be performed more than necessary.

In reality, on the other hand, because of the aging effect, accumulated degradation, and shock damages, many systems are degenerative in the sense that the successive inspection time interval will be shorter and shorter. In other words, the inter-inspection times are decreasing. In this research, we consider such situation where a geometric sequence approach is applied. The inspection times sequence \{I_1, ..., I_i, ...\} and a PM threshold level L are two important factors to be considered as decision variables for minimizing the expected long-run average cost rate. We develop a condition-based maintenance model for selecting the optimal inspection schedule and the PM threshold L for a single-unit system in order to balance the cost among PM, CM, inspection and losses due to idle time. In section 2, we describe the model assumptions and the inspection-maintenance policy. Sections 3 and 4 present a mathematical formulation for the cost rate model and the model optimization, respectively. Section 5 provides numerical examples to illustrate the results and finally conclude in section 6.

### Notation

- \( Y(t) \) Degradation value
- \( D(t) \) Cumulative shock damage value up to time \( t \)
- \( G \) A critical value for degradation process
- \( S \) A critical value for shock damage
- \( C_i \) Cost per inspection
- \( C_c \) Cost per CM action
- \( C_p \) Cost per PM action
- \( C_m \) Loss per unit idle time
- \( C(t) \) Cumulative maintenance cost up to time \( t \)
- \( E[C_i] \) Average total maintenance cost during a cycle
- \( E[W_i] \) Mean cycle length
- \( EC(I_i,L) \) Expected long-run cost rate function
- \( L \) PM critical value
- \( E[N_i]\) Number of inspection number during a cycle
- \( N_f(t) \) Number of PM actions up to time \( t \)
- \( N_c(t) \) Number of CM actions up to time \( t \)
- \( N_i(t) \) Number of inspection in \([0,t]\)
- \( \xi(t) \) Cumulative idle times in \([0,t]\)
- \( E[I]\) Mean idle time during a cycle
- \( I_{i+1} \) Inspection sequence
- \( I_i \) Inspection sequence \( i \)
- \( N_{i+1} \) Inter-inspection sequence
- \( N_{i} \) Renewal times, \( N_{i} \) first renewal time
- \( T \) Time to failure
- \( P_{i+1} \) Probability that there are total \( i+1 \) inspections in a renewal cycle
- \( P_p \) Probability that a renewal cycle ends by a PM action
- \( P_c \) Probability that a renewal cycle ends by a CM action

### 2. Model Description and Assumptions

#### 2.1 Assumptions

The system starts at a new condition. The assumptions are as follows:

1. The system is not continuously monitored, but its can be detected only by inspection. Inspections are assumed instantaneous, perfect and non-destructive.

2. The system failure is only detected by inspection. Therefore, if the system fails, it remains failed until the next inspection which causes a loss of \( C_m \) per unit time. The system is then correctly replaced.

3. PM or CM will restore the system state to a as-good-as-new state.

4. As to the cost parameters, it is assumed that corrective maintenance is more costly than a PM. And, a PM costs much more than an inspection. That implies \( C_c > C_p > C_i \).

5. \( Y(t) \) and \( D(t) \) are two random variables and are independent.
2.2. Inspection -Maintenance Policy

In this research, the system is proposed to be inspected at times $I_1, I_2, \ldots, I_n$. As the system ages, the more frequent inspection is needed. Hence, inspection intervals between two successive inspections become shorter and shorter. A geometric sequence is suitable for such situation. Therefore, the inspection sequence can be constructed as $I_n = \sum_{i=1}^{n} \alpha^{i-1} I_1$, where $0 < \alpha < 1$ and $I_1$ is the first inspection time. The inter-inspection interval can be written as $U_n = I_n - I_{n-1} = \alpha^{n-1} I_1$, where $[U_i]_{i=1}^{\infty}$ is a decreasing geometric sequence.

According to the state detected at each inspection epoch, one has to take one of the following actions:

1. If $Y(I_1) \leq L$ and $D(I_1) \leq S$, since the system is still at a good condition, we do nothing but determine the time to next inspection: $I_{n+1} = I_n + U_n$, $i=1, \ldots$ where $U_n$ is inter-inspection time between $n^{th}$ and $n+1^{th}$ and inspection.

2. If $L < Y(I_1) \leq G$ and $D(I_1) \leq S$, or the system is in a PM state and a PM is triggered. In other words, the system is not failed but already exceeded the PM threshold, a PM is performed.

3. If $Y(I_1) \leq L \cap D(I_1) > S$ or $Y(I_1) > G \cap D(I_1) \leq S$, the system is in a CM state and a CM is triggered. In other words, if the inspection reveals that the system has failed; then, a CM action is performed on the system.

It should be noted that the inspection sequence $\{I_1, I_2, \ldots, I_n\}$ is a function of $I_1$ and $\alpha$ where $\alpha$ is given. Therefore, an interesting optimization decision-making problem here is to determine $I_1$ and $L$ that minimizes the average long-run cost per unit time. After a PM or CM, the system renews. Obviously, then a new sequence of inspections can begin which is defined in the same way.

3. Maintenance Cost Analysis

In this section, an explicit expression for the average long-run maintenance cost per unit time is derived which aims at optimizing the maintenance policy through finding both PM critical threshold value $L$ and inspection sequence. Suppose that the time horizon is infinity. The total maintenance cost up to time $t$ can be defined as:

$$C(t) = C_1 N_1(t) + C_p N_p(t) + C_c N_c(t) + C_m \zeta(t) \quad (1)$$

According to the basic renewal theory, $$\lim_{t \to \infty} \frac{C(t)}{t} = E[C_1]$$

Therefore, we will study the average total maintenance cost per unit time on a single renewal cycle instead of $\lim_{t \to \infty} \frac{C(t)}{t}$. We first need to derive the cost function $E[C_1]$.

The mean total maintenance cost during a cycle $E[C_1]$ can be expressed as:

$$E[C_1] = C_1 E[N_1] + C_p P_p + C_c P_c + C_m \zeta \quad (2)$$

where $C_1$ represents the cost associated with each inspection; $C_p$ represents each PM cost; $C_c$ represents the CM cost (since failure is only found through inspection, if it occurs at instant $T$ within the inspection time interval $[I_1, I_1+1]$), the system will remain idle during the interval $[T, I_{i+1}]$ and $C_m$ is the penalty cost per unit time associated with such event.

In the following, we will perform the analytical analysis of the components in $E[C_1]$. 

![Fig. 1: The diagram of the possible renewal cycle](image-url)

1) Let $E[N_1]$ denote the mean inspection times during a cycle. $E[N_1]$ can be obtained as:

$$E[N_1] = \sum_{i=0}^{t} (i+1) P\{N_1 = i + 1\} \quad (3)$$

There will be a total of $(i+1)$ inspections during a cycle if the first time to trigger a PM or CM action within the time interval $[I_i, I_{i+1}]$. In other word, the inspection will stop when the current inspection
finds that a PM or CM condition satisfied while this situation is not revealed in the previous inspection. Let \( P[N_i = i + 1] \) be the probability that a total of \((i+1)\) inspections occurred in one cycle. Therefore,

\[
P_{i+1} = P[N_i = i + 1] = \bigcup_{j=1}^{i} P[E_j]
\]

where

\[
E_j = \{ Y(I_j) \leq L, D(I_j) \leq S \}
\]

and

\[
E_k = \{ Y(I_k) \leq L, D(I_k) \leq S \}
\]

in the time interval \([I_k, I_{k+1}]\) which represents the functioning state of the system. That is

\[
P\bigg( \bigcup_{j=1}^{i} E_j \bigg) = P\{ Y(I_j) \leq L, D(I_j) \leq S \}
\]

Therefore,

\[
P_{i+1} = P\bigg( \bigcup_{j=1}^{i} E_j \bigg) = P\{ Y(I_k) \leq L, D(I_k) \leq S \} - P\{ Y(I_{i+1}) \leq L, D(I_{i+1}) \leq S \}
\]

(4)

Since a PM or CM action appears only once during a cycle, either of them will end a renewal cycle. As a consequence, \( P_p + P_c = 1 \). In other words, both PM and CM events are mutually exclusive at a renewal time point. \( P_p \) is defined as:

\[
P_p = P[\text{a PM action ends cycle | cycle is over}]
\]

\[
= \sum_{i=0}^{\infty} P[\text{a PM action ends cycle}, N_i = i + 1 | \text{cycle is over}]
\]

\[
= \sum_{i=0}^{\infty} P_{i+1}(E_i)
\]

(5)

where.

\[
P_{i+1}(E_i) = P\{ Y(I_i) \leq L, L < Y(I_{i+1}) \leq G \} \cdot P\{ D(I_{i+1}) \leq S \}
\]

Since PM and CM are mutually disjointed at renewal time, \( P_c = 1 - P_p \).

To compute \( P[Y(l) \leq L, L < Y(I) \leq G] \) while \( Y(l) \) are \( Y(I_{i+1}) \) dependent, we need to take the joint p.d.f \( f_{Y(I),Y(I_{i+1})}(x_1, x_2) \) of \( Y(I) \) and \( Y(I_{i+1}) \). We now explore the following two different degradation functions for \( Y(t) \):

1. Assume \( Y(t) = A + B \cdot I(t) \) where \( A \) and \( B \) are independent. Assume that \( A \sim f_A(a), B \sim f_B(b) \).

Let

\[
\begin{align*}
\gamma_1 &= a + bg(I_i) \\
\gamma_2 &= a + bg(I_{i+1})
\end{align*}
\]

After solving the above equations in terms of \( \gamma_1 \) and \( \gamma_2 \), we have:

\[
a = \frac{y_1 g(I_{i+1}) - y_2 g(I_i)}{g(I_{i+1}) - g(I_i)} \quad \text{and} \quad b = \frac{y_2 - y_1}{g(I_{i+1}) - g(I_i)}
\]

Suppose \( y_1, y_2, h_1, h_2, h_3, h_4 \) are continuous and the partial derivatives \( \frac{\partial h_1}{\partial y_1}, \frac{\partial h_1}{\partial y_2}, \frac{\partial h_2}{\partial y_1}, \frac{\partial h_2}{\partial y_2} \) are differentiable. Then the Jacobian \( J \) is given by

\[
J = \begin{vmatrix}
\frac{\partial h_1}{\partial y_1} & \frac{\partial h_1}{\partial y_2} \\
\frac{\partial h_2}{\partial y_1} & \frac{\partial h_2}{\partial y_2}
\end{vmatrix} = \frac{1}{g(I_i) - g(I_{i+1})}.
\]

Then the random vector \( (Y(I), Y(I_{i+1})) \) has a joint continuous probability density function (pdf) given by:

\[
f_{Y(I),Y(I_{i+1})}(y_1, y_2) = |J| f_A(h_1(y_1, y_2)) f_B(h_2(y_1, y_2))
\]

(6)

2. Assume \( Y(t) = \lambda e^{-\lambda t} \) where \( A \) and \( B \) are independent.

Assume \( A \sim f_A(a), B \sim f_B(b) \).

Let

\[
\begin{align*}
y_1 &= \frac{\lambda e^{-\lambda t}}{b + e^{-\lambda t}} \\
y_2 &= \frac{\lambda e^{-\lambda t}}{b + e^{-\lambda t}}
\end{align*}
\]
The above equations can be uniquely solved for $a$ and $b$ in terms of $y_1$ and $y_2$ with solutions given by

$$
\begin{align*}
    a &= \ln \left( \frac{y_1(y_1-1)}{y_2(y_2-1)} \right) / (l_{i+1} - l_i) = h_1(y_1, y_2) \\
    b &= -e^{\ln(y_2(y_2-1)/y_1(y_1-1))} (l_{i+1} - l_i) = h_2(y_1, y_2)
\end{align*}
$$

Suppose that the functions $h_1(y_1, y_2)$ and $h_2(y_1, y_2)$ have continuous partial derivatives. The following is a $2 \times 2$ Jacobian determinant:

$$
\begin{align*}
    j &= \left| \begin{array}{cc}
        d_1(y_1, y_2) & d_2(y_1, y_2) \\
        d_3(y_1, y_2) & d_4(y_1, y_2)
    \end{array} \right| \\
    d_1(y_1, y_2) &= e^{\ln(y_2(y_2-1)/(y_1(y_1-1)))/l_{i+1} - l_i} \\
    d_2(y_1, y_2) &= e^{\ln(y_2(y_2-1)/(y_1(y_1-1)))/l_{i+1} - l_i} (y_2 - 1) \\
    d_3(y_1, y_2) &= \frac{y_1 - 1}{y_1(y_1-1)} - \frac{y_2(y_1-1)}{y_1(y_2-1)^2} y_1^2 (y_2 - 1)^3 \\
    d_4(y_1, y_2) &= \frac{y_2}{y_1 - 1} - \frac{y_2(y_1-1)}{y_1(y_2-1)^2} y_1^2 (y_2 - 1)^3 \\
    d_5(y_1, y_2) &= \frac{d_3(y_1, y_2)}{y_2^2 (y_1 - 1)^2 (l_{i+1} - l_i)^2}
\end{align*}
$$

For example, given $A \sim U[0, a]$ and $B \sim Weibull(\eta, \beta)$.

The joint pdf is given by

$$
f_{Y(I,Y(I_{i+1}))}(y_1, y_2) = \int f_A(h_1(y_1, y_2)) f_B(h_2(y_1, y_2))
$$

(7)

If $0 < \frac{l_{i+1} - l_i}{y_2(y_2-1)/y_1(y_1-1)} < a$, we have the path how to compute $P\{Y(I_i) \leq L, L < Y(I_{i+1}) \leq G\}$.

3) When $I_i < T \leq I_{i+1}$ the unit will be idle during the interval $[T, I_{i+1}]$. Let $E[\zeta]$ denote the average idle time between the failure occurrence epoch and its inspection during one cycle. $E[\zeta]$ is calculated as follows:

$$
E[\zeta] = \sum_{i=0}^{\infty} \left( \sum_{j=2}^{\infty} p_{i,j} \left( F(t) \right) \int_{t_i}^{t_{i+1}} f_r(t) dt \right)
$$

(9)

where $f_r(t) = \frac{d}{dt} (F(t))$, is the p.d.f of $T$ and

$$
F(t) = P\{Y(t) \leq G, D(t) > S\} + P\{Y(t) > G, D(t) \leq S\}
$$

Next we will need to obtain the expected cycle length function. The mean cycle length $E[W_i]$ is calculated as follows:

$$
E[W_i] = E[E[W_i | N_i]]
$$

(10)

It can be shown that the random vector $(Y(I_i), Y(I_{i+1}))$ are jointly continuous with a joint density function given by:
EC is a function of inspection times \([I_{t_1}, I_{t_2}, \ldots, I_{t_n}]\) and PM threshold \(L\) through, \(P_x\), \(P_y\), \(E[N_x]\), \(E[S]\) and \(E[W]\). We then, in section 4, determine the optimal inspection sequence \([I_{t_1}, \ldots, I_{t_n}]\) and PM threshold level \(L\) which minimizes the long run average cost per unit time, \(EC\).

4. Optimization Maintenance Policy

In section 3, we discuss each component of the expected long-run maintenance cost rate. In this section, we determine \(I_i\) and \(L\) so that the expected long-run maintenance cost rate is minimized. The following optimization problem is formulated in terms of decision variables \(I_i\) and \(L\) (0 < \(L\) \(\leq\) \(G\))

\[
\min_{EC(I_i, L)} = \frac{C_i}{\sum_{t=1}^{n} t} + \frac{C_e}{\sum_{t=1}^{n} (t+1)} \left[ P(Y(t) \leq L, D(t) \leq S) - P(Y(t) \leq L, D(t) > S) \right]
\]

\[
+ \frac{C_c}{\sum_{t=1}^{n} t} C_c \left[ P(Y(t) \leq L, D(t) \leq S) - P(Y(t) \leq L, D(t) > S) \right]
\]

\[
+ \frac{C_p}{\sum_{t=1}^{n} t} C_p \left[ P(Y(t) \leq L, D(t) \leq S) - P(Y(t) \leq L, D(t) > S) \right]
\]

\[
+ \frac{C_m}{\sum_{t=1}^{n} t} C_m \left[ P(Y(t) \leq L, D(t) \leq S) - P(Y(t) \leq L, D(t) > S) \right]
\]

(11)

It is difficult to obtain the closed-form expression for the optimal inspection-maintenance policy \((I^{*}_{t_1}, L^{*})\) that minimizes the equation (11). Since this is a two-dimension decision variable, one can use the traditional optimization method to obtain a local minimal optimum solution. General idea is to fix one dimension such as \(I_{t_1}\) and search for a local minimal value \(L\) and so forth, given that one needs to provide the initial value for \(L\). However, depend on the initial value of \(L\), the local solution may still not closed enough to the optimum solution. To improve the search for obtaining a closed-best solution, we use the idea in [20] by considering the mean time to first failure as an initial solution for \(I_{t_1}\) to begin with for searching the optimum solution. After several trials and numerous calculations compared between the traditional optimization method (that is, without the knowledge of the mean time to first failure calculation) and our proposed procedure, we observe that our approach converges quickly to the “best” local solution, if not global solution. Below is our proposed algorithm that uses to obtain the “best” local solution for the proposed model in equation (11).

Algorithm:

Step 1: Initialize the cost parameters \(C_x, C_y, C_p\) and \(C_e\). Let initial \(I_i = E[T]\), where

\[
E[T] = \int_{0}^{\infty} P[T > t] dt = \int_{0}^{\infty} P[Y(t) \leq G, D(t) \leq S] dt
\]

(12)

Step 2: Search one-dimensional minimal value along \(L\) direction and record it as \(EC(I^{*}_{t_1}, L^{*})\)

Step 3: Increase \(I_{t_1}\) value. Let \(I_{t_1} = I_{t_1} + \Delta\) where \(\Delta\) is a small increment.

Step 4: Search new one-dimensional minimal value along \(L\) direction and record it as \(EC(I^{*}_{t_1}, L^{*})\)

Step 5: If \(EC(I^{*}_{t_1}, L^{*}) > EC(I^{*}_{t_1}, L^{*})\) then \(EC(I^{*}_{t_1}, L^{*}) = EC(I^{*}_{t_1}, L^{*})\) and go to Step 3. If \(EC(I^{*}_{t_1}, L^{*}) > EC(I^{*}_{t_1}, L^{*})\) then \(EC(I^{*}_{t_1}, L^{*}) = EC(I^{*}_{t_1}, L^{*})\) and go to Step 6.

Step 6: Stop.

5. Numerical Examples

For the degradation function \(Y(t) = A + Bg(t)\) assuming \(A \sim U[0, 2]\), \(B \sim Exp(-0.2)\) and \(g(t) = I_{t}e^{-0.01t}\). For the random shock damage \(D(t) = \sum_{i=1}^{N_{t}} X_i\), assuming that \(X_i \sim Exp(-0.04t)\) and \(N(t) \sim Poisson(0.1)\). Given \(G = 50\), \(S = 100\), \(a = 0.98\) and the cost parameters as: \(C_x = 200\) per inspection, \(C_p = 5600\) per CM, \(C_y = 3000\) per PM, \(C_m = 700\) per unit time.

The inspection sequence \([I_{t_1}, \ldots, I_{t_n}]\) is constructed using the formula \(I_{t} = \sum_{i=0}^{n} \alpha^{i-1}I_{t_1}\), where \(\alpha = 0.8\). We now determine both the value of \(L\) and \(I_{t_1}\) so that the expected total cost per unit time, \(EC(I_{t_1}, L)\), is minimized. We first need to calculate the mean time to first failure and from equation (12), we obtain \(E[T] = 40.8\). Therefore, an initial value for \(I_{t_1}\) is \(I_{t_1} = 40.8\).

Figure 2 pictures a 3D long-run average total cost rate in terms of \(L\) and \(I_{t_1}\). The optimum solutions for the PM threshold level and inspection sequence are \(I_{t_1} = 58\) and \(L = 28\), respectively, and the corresponding expected long-run cost rate is \(EC(I_{t_1}, L) = 94.63\) where the average total maintenance cost during a cycle and the mean cycle length are, respectively, \(E[C] = 5685.13\) and \(E[W] = 60.07\). The optimal results balance the cost of inspections, preventive maintenance, corrective maintenance and penalty associated with idle time.
Cost during a cycle and the mean cycle length are, as L increases since $\sum_{i=1}^{\infty} \frac{1}{i} = \infty$. Consequently, $P_c = 1 - \sum_{i=1}^{\infty} \frac{1}{i}$ is also a decreasing function of L. Consequently, it can be shown that

$$P(\xi(1) \leq L, L < \xi(1) \leq G)$$

is a decreasing function of L as L increases. Therefore, it can be shown that

$$P_p = \sum_{i=1}^{\infty} P_i(\xi_i) = \sum_{i=1}^{\infty} P(\xi(1) \leq L, L < \xi(1) \leq G | \xi(1) \leq S)$$

is also a decreasing function of L. Consequently, $P_p$ increases as L increases since $P_c = 1 - P_p$. From the figure 3, we easily observe that $P_p$ is a decreasing function of $L$.

Figure 4 pictures the expected idle time during a cycle $E[\xi]$ versus the PM critical values of $L$ for various inspection sequences such as: $I_1 = 62$, $I_1 = 60$, $I_1 = 58$ and $I_1 = 56$.

Since

$$P(\xi(1) \leq L, L < \xi(1) \leq G | \xi(1) \leq S)$$

is an increasing function as L increases, it can be shown that the mean idle time during a cycle $E[\xi]$ is an increasing function of $L$.

6. Conclusion

This paper we present a condition-based inspection-maintenance model for degraded systems with respect to degradation process and random shocks.
based on geometric sequences for inspection intervals. We determine the optimum decision variables for the PM threshold level $L$ and the inspection sequence that minimizes the expected long-run cost rate. The difference between the proposed condition-based inspection/maintenance policy and a classical periodic inspection is that the periodic inspection times which constitutes a shorter and shorter inter-inspection sequence, instead of a larger inter-inspection sequence [20]. This sequence brings realistic implementations to several critical applications in practices using geometric sequence approach. The proposed policy is based on the measurable amount of the degradation and shock damage instead of assuming the processes modeled by Markov, semi-Markov process or imposing some constraints on the process in order to make the model computable. These advantages make our model practicable and interesting.

References